

A Check-up for the Statistical Parton Model

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Abstract. We compare the parton distributions deduced in the framework of a quantum statistical approach for both the longitudinal and transverse degrees of freedom with the unpolarized distributions measured at Hera and with the polarized ones proposed in a previous paper, which have been shown to be in very good agreement also with the results of experiments performed after that proposal. The agreement with Hera data in correspondence of very similar values for the "temperature" and the "potentials" found in the previous work gives a robust confirm of the statistical model. The feature of describing both unpolarized and polarized parton distributions in terms of few parameters fixed by data with large statistics and small systematic errors makes very attractive the parametrization proposed here.

PACS. 12.40.Ee : Statistical Model , 14.65.Bt : Light Quarks
13.60.Hb : Total and inclusive cross sections (including deep-inelastic processes)

1 Introduction

About twenty years ago [1] the similar shapes of the polarized structure function $xg_1^p(x)$ and of the difference $F_2^p(x) - F_2^n(x)$ suggested that for the shapes and the first moments of the valence quark parton distributions there is the correlation dictated by Pauli principle, which also accounts for the defect in the Gottfried sum rule [2] found experimentally [3], related to the isospin asymmetry $\bar{d} - \bar{u}$ advocated since a long time [4]. The role of Pauli principle is a robust motivation to write for the distributions of the valence partons Fermi-Dirac functions in the variable x , which appears in the parton model sum rules.

The shape-first moment correlation implied by Pauli principle accounts for the dramatic high x decrease of the ratio $F_2^n(x)/F_2^p(x)$ and for the increasing (decreasing) x -dependance of the positive (negative) ratio $\Delta u(x)/u(x)$ ($\Delta d(x)/d(x)$) [5]. After many attempts a satisfactory description of a selected choice of precise unpolarized and polarized structure functions has been obtained [6] by adding to a Fermi-Dirac expressions for the quark partons an unpolarized and iso-scalar term, which may be interpreted as the gluon contribution at order α_s , with a power in front of the FD functions multiplied by a constant proportional to the "potential" associated to each parton for the valence quarks. A crucial role to fix the shape of the gluon distribution (evidently a Bose-Einstein function in the x variable) is plaid by the equilibrium conditions for the elementary QCD processes (emission of a gluon by a quark and conversion of a gluon in a $q\bar{q}$ pair), which imply a zero value for the potentials of the gluons with both helicities (Bose-Einstein turns

into Planck and $\Delta G(x) = 0$) and opposite values for the valence quarks and their antiparticles with opposite helicity. In [6], we have been able to describe both unpolarized and polarized distributions in terms of few parameters and in subsequent works, we compared our predictions with experimental results obtained in the following years [7, 8] in agreement both for the polarized structure functions $g_1^{p,n,d}$ and for the unpolarized structure functions measured in the electromagnetic and weak DIS at Hera. Also we tried to explain the "ad hoc" factors X_q^h , we had to introduce for the non-diffractive part of the fermion parton distributions. We realized that with the extension of the statistical approach to the transverse degrees of freedom from a sum rule for the transverse energy one is able to predict a gaussian dependance on p_T with a width increasing with x and fix the "transverse potentials" to reproduce the "ad hoc" factors introduced for the valence partons in [6]. The purpose of this paper is to provide a general check of the approach proposed in [6]. In the following section, we recall the expressions for the partons introduced in [6]. In section 3, the consequences of the extension to the transverse momenta are described, by keeping also into account of Melosh rotation [9]. In Section 4, our expressions are compared with the light parton distributions obtained in the combined fit to $H_1 - Zeus$ data performed at Hera [10] for the unpolarized distributions and with the expressions for the polarized distributions given in [6] and very successful to describe the measurements depending on them after 2002 [7, 8, 11]. In Section 5, we give our conclusions.

2 The Statistical Model Proposed in 2002

Let us recall the expressions and the values of the parameters, which allowed to get a fair description of both unpolarized and

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polarized structure functions [6]. For the light valence partons at $Q^2 = 4(\text{GeV}^2)$, we assumed:

$$xu^+(x) = \frac{Ax^b X_{u^+}}{\exp[(x - X_{u^+})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \quad (1)$$

and their antiparticles:

$$x\bar{u}^+(x) = \frac{\tilde{A}x^{2b}(X_{u^-})^{-1}}{\exp[(x + X_{u^-})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \quad (2)$$

and corresponding expressions for u^- , d^+ , d^- and their antiparticles with $X_{u^+} \rightarrow X_{u^-}$, X_{d^+} and X_{d^-} , respectively for eq.(1), and $X_{u^-} \rightarrow X_{u^+}$, X_{d^-} and X_{d^+} for eq.(2)

For gluons one assumed the Bose-Einstein form:

$$xG^\pm(x) = \frac{A_g x^{b_g}}{\exp(x/\bar{x}) - 1} \quad (3)$$

The opposite values for the potentials X_{u^+} and X_{u^-} and the vanishing value for X_{G^+} and X_{G^-} , which turns the Bose-Einstein into a Planck expression, follow from the equilibrium condition with respect to the elementary QCD processes [12]:

$$q^\pm \rightarrow q^\pm + G \quad (4)$$

$$G \rightarrow q^{(\pm)} + \bar{q}^{(\mp)} \quad (5)$$

The coefficients in the first term of eq.1 were introduced to agree with data and, just by guess, we assumed the same coefficient in the denominator in eq.2. The interpretation of the second term in eqs.(1) and (2) as the diffractive contribution coming from the gluons lead to the assumption:

$$b_g = \tilde{b} + 1 \quad (6)$$

The constraints:

$$u - \bar{u} = 2 \quad (7a)$$

$$d - \bar{d} = 1 \quad (7b)$$

and the requirement that the partons carry the longitudinal momentum of the proton fix A , \tilde{A} and A_g .

The 2002 fit was obtained with the following values of the parameters:

$$\begin{aligned} \bar{x} &= 0.09907, X_{u^+} = 0.46128, X_{d^-} = 0.30174, X_{u^-} = 0.29766 \\ X_{d^+} &= 0.22775, A = 1.74938, \tilde{A} = 1.90801, A_g = 20.53 \\ \tilde{A} &= 0.08318, b = 0.40962, \tilde{b} = -0.25347 \end{aligned} \quad (8)$$

3 The Extension of the Statistical Approach to the Transverse Degree of Freedom

In [7], the predictions of the statistical model introduced in [6] have been compared with measurements performed after with a particular success for the polarized structure function $xg_1^n(x)$, expected to be negative at small x and positive at large x and with a good precision for the value of x , where it vanishes. In [8] a successful test of the predictions has been performed for the unpolarized structure functions for electromagnetic and weak DIS scattering at Hera. Then to account for the "ad hoc" factors introduced for the non-diffractive part of the valence parton distributions and for their antiparticles, a sum rule has been assumed for the transverse energy [11], which fixes in terms of a dimensional Lagrange multiplier $1/\mu^2$ and of the transverse potentials, \tilde{Y}_q^h , the p_T^2 dependance, which tends for large p_T^2 to a gaussian form, by other authors considered without a justification, with a width proportional to \sqrt{x} .

The equations for the parton distributions depending on x and p_T^2 for the non-diffractive part of the unpolarized distributions are:

$$\begin{aligned} xu(x, p_T^2) &= \frac{1}{x\mu^2} \left(\frac{A'x^b}{e^{(x-\tilde{X}_u^+)/\bar{x}} + 1} \frac{1}{e^{(2p_T^2/\mu^2(x+\sqrt{x^2+(p_T^2/P_z^2)}))-\tilde{Y}_u^+} + 1} \right. \\ &\quad \left. + \frac{A'x^b}{e^{(x-\tilde{X}_u^-)/\bar{x}} + 1} \frac{1}{e^{(2p_T^2/\mu^2(x+\sqrt{x^2+(p_T^2/P_z^2)}))-\tilde{Y}_u^-} + 1} \right) \end{aligned} \quad (9)$$

$$\begin{aligned} xd(x, p_T^2) &= \frac{1}{x\mu^2} \left(\frac{A'x^b}{e^{(x-\tilde{X}_d^+)/\bar{x}} + 1} \frac{1}{e^{(2p_T^2/\mu^2(x+\sqrt{x^2+(p_T^2/P_z^2)}))-\tilde{Y}_d^+} + 1} \right. \\ &\quad \left. + \frac{A'x^b}{e^{(x-\tilde{X}_d^-)/\bar{x}} + 1} \frac{1}{e^{(2p_T^2/\mu^2(x+\sqrt{x^2+(p_T^2/P_z^2)}))-\tilde{Y}_d^-} + 1} \right) \end{aligned} \quad (10)$$

and similar expressions for $x\Delta u$ and $x\Delta d$ except the minus sign instead of the positive sign between the two terms in the right hand side of eqs.(9) and (10) and also, the factor coming from Melosh transformation.

For the antiquarks, one has the expressions in which the \tilde{X} 's and the \tilde{Y} 's have opposite sign than their antiparticles with opposite and a different normalization constant, \tilde{A} helicity. For the gluons, we write the expression:

$$xG(x) = \frac{A_g x^{b_g} e^{-C_g(x+c_g)^2}}{e^{(x/\bar{x})} - 1} \left(1 + \ln\left(\frac{x+r}{x+u}\right) \right) \quad (11)$$

where the two slowly varying factors, which slightly modify the behavior dictated by the Planck expression for gluons, are needed to agree almost perfectly with [10].

By integrating in dp_T^2 eqs.(9) and (10) one gets:

$$\begin{aligned} xq(x) &= \frac{A'x^b}{e^{(x-\tilde{X}_q^+)/\bar{x}}} \left[\ln(1 + e^{\tilde{Y}_q^+}) + \frac{2\mu^2(1-x)}{Q^2} (-Li_2(-e^{\tilde{Y}_q^+})) \right] \\ &\quad + \frac{A'x^b}{e^{(x-\tilde{X}_q^-)/\bar{x}}} \left[\ln(1 + e^{\tilde{Y}_q^-}) + \frac{2\mu^2(1-x)}{Q^2} (-Li_2(-e^{\tilde{Y}_q^-})) \right] \end{aligned} \quad (12)$$

$$x\Delta q(x) = \frac{A'x^b}{e^{(x-\tilde{X}_q^+)/\bar{x}}} \left[\ln(1 + e^{\tilde{Y}_q^+}) \right] - \frac{A'x^b}{e^{(x-\tilde{X}_q^-)/\bar{x}}} \left[\ln(1 + e^{\tilde{Y}_q^-}) \right] \quad (13)$$

Where $q = u, d$ and the small correction proportional to the ratio $2\mu^2/Q^2$ is absent for the polarized distributions since it is exactly compensated by the consequence of the Melosh-Wigner rotation [9].

In the limit of large Q^2 we get the factor $\ln(1 + \exp[\tilde{Y}_q^h])$, which we can reasonably assume to be proportional to \tilde{X}_q^h (valence quark with larger first moments are expected to have broader shapes both in the longitudinal and transverse degrees of freedom). While with the above assumption we can get the "ad hoc" factors for valence partons, for their antiparticles a similar property can be obtained only approximately. In fact, while we guessed in [6] the product of the factors for the valence quark of given helicity and of their antiparticle with opposite helicity to be constant, the function:

$$\ln(1 + \exp[\tilde{Y}])\ln(1 + \exp[-\tilde{Y}])$$

has its maximum $(\ln 2)^2$ at $\tilde{Y} = 0$, and therefore the assumption of the proportionality between $\ln(1 + \exp[\tilde{Y}_q^h])$ and \tilde{X}_q^h implies a smaller coefficient for the more rare light antipartons with the consequence that with the same \tilde{X}_q^h for the non-diffractive part the ratio $\Delta\bar{u}/\bar{u}$ becomes more positive and $\Delta\bar{d}/\bar{d}$ more negative.

If we apply to the gluons the extension to the transverse degrees of freedom, with a vanishing transverse potential for both helicities, we find a divergent expression, but we can avoid the inconvenient by observing that with $b_g = 1$, as we impose according to the radiation interpretation of the gluon component, its contribution to the longitudinal and transverse energy sum rules, albeit divergent, are in the ratio μ^2/\bar{x} . Since for the gluon we did not introduce, as for the fermionic partons, arbitrary factors, we can take for the gluon distribution the form assumed in eq.(11) with $b_g = 1$ and assume for the contribution to the transverse energy [11] of the gluon the value given by the product of its contribution to the longitudinal sum rule times μ^2/\bar{x} .

Another difference with respect to [6] is that the Melosh rotation [9] for the polarized distributions implies a very small difference for the normalization of the polarized and unpolarized distributions.

4 The Comparison with Hera Data for the Statistical Parton Model

We want to test the statistical parton model by describing with the form given in the previous section for the unpolarized light fermion partons with the result of the combined $H_1 - ZEUS$ fit [10]. As long as the polarized distributions of the light partons, the success in describing the $xg_1^{p,n,d}$ structure functions and the confirm [11] of our prediction of a positive value for $\Delta\bar{u}$ and a negative for $\Delta\bar{d}$ gives us the motivation to require that the polarized distributions are just the ones found in [6]. With respect to [6] article we assume the same exponent, b , also for

the non-diffractive part of the light anti-quarks and we fix b_g to be 1, which is the value expected for a radiation term. The extension to the transverse degrees of freedom implies a small difference for the unpolarized and polarized distributions as a consequence of the Melosh [9] rotation. Also, the proportionality between:

$\ln(1 + \exp[\tilde{Y}_q])$ and \tilde{X}_q does not imply the proportionality between $\ln(1 + \exp[-\tilde{Y}_q])$ and $1/\tilde{X}_q$ with the consequence that for the non-diffractive part of the antiquarks with \tilde{Y}_q^h given by eq.11 one has a more negative value for $\Delta\bar{d}(x)/\bar{d}(x)$ and a more positive value for $\Delta\bar{u}(x)/\bar{u}(x)$. In conclusion the formulas have the following differences from [6]:

i) Instead of the "ad hoc" factors \tilde{X}_q^h for valence quarks we have the factors $\ln(1 + \exp[\tilde{Y}_q^h])$ depending on the transverse potentials, while for their antiparticles with opposite helicity we have the factors $\ln(1 + \exp[-\tilde{Y}_q^h])$.

ii) By keeping into account of the Melosh rotation [9], we have the difference between the polarized and the unpolarized distribution with the additional factor for these proportional to $\frac{2\mu^2(1-x)}{Q^2}$, which anyway gives a small correction, since μ^2 is about $0.1(\text{GeV})^2$ and $Q^2 = 4(\text{GeV})^2$.

iii) We take the same exponent, b , for the power for the light anti-quarks, and fix to $b_g = 1$, the exponent for the gluon, as it is appropriate for a radiation term.

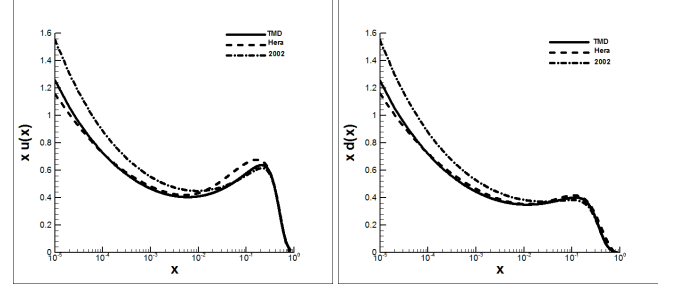


Fig. 1. The distribution of quarks up and down in comparison to the 2002 results and Hera data.

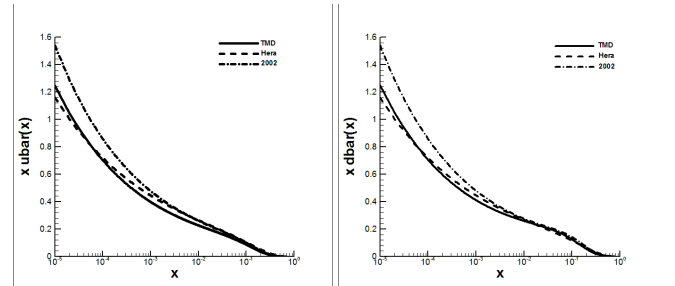


Fig. 2. The distribution of anti- quarks upbar and downbar in comparison to the 2002 results and Hera data.

In figures (1) and (2), we compare the expressions given in the previous section with the determination of HERA through the combined fit $H_1 - ZEUS$ of the light valence partons and their antiparticles. The success in the comparison with exper-

iment of the polarized structure functions [7, 8] and the evidence for a positive $\Delta\bar{u}(x)$ and for a negative $\Delta\bar{d}(x)$ [11] also quantitatively as predicted in [6] motivates our demand that the polarized distributions of the light partons be equal to the ones proposed in that paper. We compare in figures 3-4 the polarized distributions with the expressions found in [6]. Finally we fix the parameters in eq (11) to agree with [10], as shown in figure 5. The values for the parameters are the following:

$$\begin{aligned}\tilde{X}_{u+} &= 0.446, \tilde{X}_{d+} = 0.222, \tilde{X}_{d-} = 0.320, \tilde{X}_{u-} = 0.297 \\ \tilde{Y}_{u+} &= 1.050, \tilde{Y}_{d+} = 0.01, \tilde{Y}_{d-} = 0.360, \tilde{Y}_{u-} = 0.293 \\ A' &= 0.615, \bar{A}' = 3.50, \mu^2 = 0.0938, b = \bar{b} = 0.430 \\ \tilde{A} &= 0.070, \tilde{b} = -0.250, \bar{x} = 0.102 \\ A_g &= 168, C_g = 2.475, c_g = 0.765, r = 0.00116, u = 0.001\end{aligned}\quad (14)$$

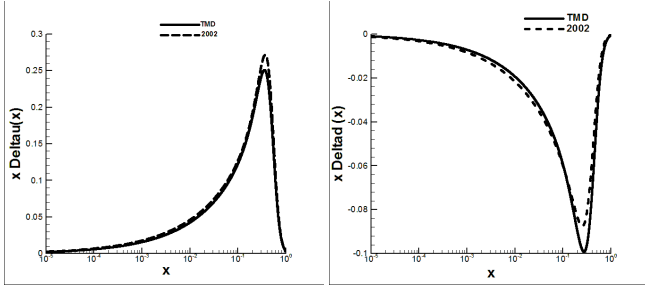


Fig. 3. The distribution of Δu and Δd in comparison to the 2002 results.

The excellent agreement shown in these figures and the agreement of the parameters with the ones found in [6] with the only exception of \tilde{A} fixed by data at small x changed since 2002 (the exponent for the non-diffractive term for the light anti-quarks has been chosen to be equal to the one for the valence partons) is a good confirm of our approach, which also implies the exponential behavior $\exp(-x/\bar{x})$ at high x shown experimentally by the distributions. As long for the gluons, the Planck expression reproduces the small x behavior and the exponential fall at large x of $xG(x)$, but needs the "ad hoc" factors introduced in (11) to get the agreement with data. The parameters found are in a very good agreement with the ones found in [6], as shown by the comparison of the parameters:

	ref. [6]	this paper	$\frac{A'}{\bar{A}} \ln(1 + \exp \tilde{Y}_q)$
\tilde{X}_{u+}	0.46188	0.446	0.4650
\tilde{X}_{d-}	0.30174	0.320	0.3115
\tilde{X}_{u-}	0.29766	0.297	0.2975
\tilde{X}_{d+}	0.22775	0.222	0.2345
b	0.40962	0.43	
\tilde{A}	0.08318	0.070	
\tilde{b}	-0.25347	-0.250	
\bar{x}	0.09907	0.102	

where we have reported in the third column the coefficients found with the extension to the transverse momenta, comparing them with the "ad hoc" factors \tilde{X}_q introduced in [6].

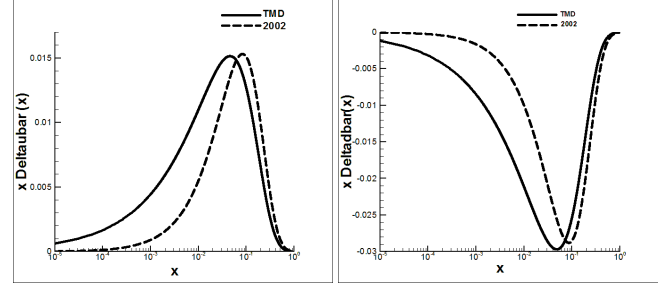


Fig. 4. The distribution of $\Delta\bar{u}$ and $\Delta\bar{d}$ in comparison to the 2002 results.

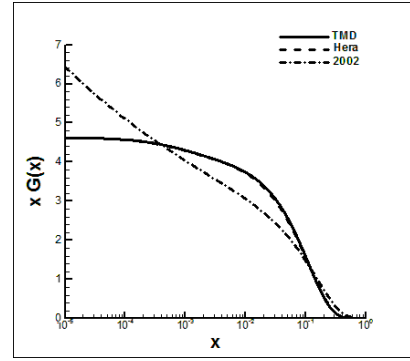


Fig. 5. The distribution of gluons in comparison to the 2002 results and Hera data.

In figures 6-7-8 the ratios $\frac{d(x)}{u(x)}$, $\frac{\Delta u(x)}{u(x)}$ and $\frac{\Delta d(x)}{d(x)}$ are compared with [6] and for the first of them also with Hera. Finally in fig.9 we compare $x\bar{d}(x) - x\bar{u}(x)$ with the expression found in [6]. The exponential behaviour $\exp(-x/\bar{x})$ predicted by the statistical model (for the \bar{q} 's with negative \tilde{X} 's with modulus larger than \bar{x} the Boltzmann limit is a good approximation) agrees with the data from the Fermilab E866 Drell-Yan experiment [14] displayed in [15], where the definitions "intrinsic" and "extrinsic" correspond here to "non diffractive" and "diffractive" terms.

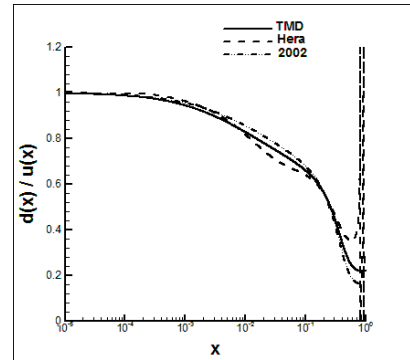


Fig. 6. The distribution of $d(x)/u(x)$ in comparison to the 2002 results and Hera data.

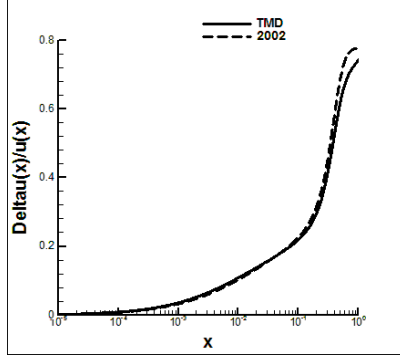


Fig. 7. The distribution of $\Delta u(x)/u(x)$ in comparison to the 2002 result.

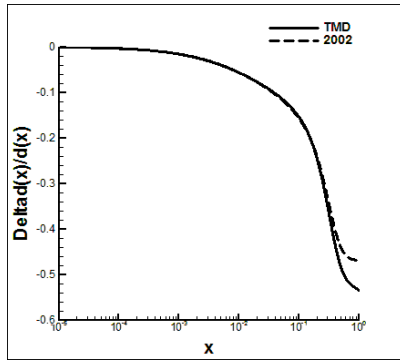


Fig. 8. The distribution of $\Delta d(x)/d(x)$ in comparison to the 2002 result.

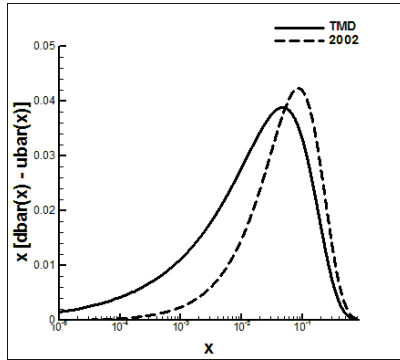


Fig. 9. The distribution of $x\bar{d}(x) - x\bar{u}(x)$ in comparison to the 2002 result.

	Hera LSR	this paper LSR	this paper TSR
<i>Gluon</i>	0.410	0.409	0.4265
<i>u</i>	0.312	0.298	0.3153
<i>d</i>	0.162	0.152	0.1758
\bar{u}	0.0313	0.0243	0.0145
\bar{d}	0.0366	0.0322	0.0377

	ref. [6]	this paper
$\int [\Delta u(x)] dx$	0.663811	0.642376
$\int [\Delta d(x)] dx$	-0.255714	-0.258968
$\int [\Delta \bar{u}(x)] dx$	0.0464154	0.0671633
$\int [\Delta \bar{d}(x)] dx$	-0.0865359	-0.13182
$\int [\bar{d}(x) - \bar{u}(x)] dx$	0.12739	0.17123

	Hera	this paper	ref. [6]	ref. [13]
$d(1)/u(1)$	0.31167	0.219354	0.159891	0.22
$\Delta u(1)/u(1)$		0.742825	0.77773	
$\Delta d(1)/d(1)$		-0.53318	-0.467774	

In the Tables we compare the contribution to the longitudinal and transverse sum rules of the fermion and of the gluon with Hera result for the longitudinal sum rule and the first moments of the polarized parton distributions with [6]. The ratios of the limits of the parton distributions at $x = 1$ are also shown and compared with [6], and also with [10] and [13] for $d(1)/u(1)$. A point in favour of our parametrization with the universal Boltzmann behaviour $\exp(-x/\bar{x})$ at high x is the fact that, despite we fixed the parameters for the unpolarized distributions in order to be as much as possible equal to [10], in the $x \rightarrow 1$ limit, where their extrapolation is less predictive, the ratio $d(1)/u(1)$ is in better agreement with [13] for the statistical distributions. With respect to [16], while $\frac{\Delta u(1)}{u(1)} + \frac{4d(1)}{3u(1)}$ is near to 1, the value expected in that paper, for $\Delta d(1)/d(1)$ we predict a value more negative than $-\frac{1}{3}$. The sum of the numbers in the third column of the second table should give 1 to obey the transverse energy sum rule (which is divided by M^2).

5 Conclusion

The purpose of this paper was to perform a check-up of the statistical parton model by taking advantage of our understanding of the transverse distributions, which improves the theoretical consistency of our statistical approach, through the comparison with the Hera fit based on the combined analysis of $H_1 - ZEUS$ data performed after ref.[6]. The fact that we have required the polarized distributions equal [6] is fully justified by the remarkable success in describing polarized data again taken after both for g_1^{p,d,He_3} [7, 8] and for the more recent W^\pm production[12]. The reader may judge by looking to the figures, we present, with the values of the parameters found so similar to the ones proposed in 2002 on the goodness of that proposal, which in our judgement may help in getting from experiment the parton distributions, in particular the polarized ones, which we have been able to well describe in their qualitative and quantitative properties. The choice of the same exponent, b , for the exponent of the power factor in the non diffractive part of the fermion parton distributions is well consistent with the Hera result[10] for the unpolarized light anti-quark distributions and implies larger polarizations for them at low x with respect to [6] to be compared with experiment in that region, a more positive contribution to the Bjorken sum rule [17] and a larger negative contribution to the Gottfried sum rule [2]. As long as for gluons the Planck form $xG(x)$ proportional to $\frac{x}{e^{x/\bar{x}} - 1}$ looks like the Hera result with a rather flat behavior at small x and the exponential behavior at high x with the same exponent of the valence partons, but to coincide with it needs the slowly varying factor written in eq.(11), for which at the moment we are not able to give an interpretation. The p_T dependance implied by the transverse sum rule for p_T^2 larger than $\mu^2 x \bar{Y}_q$ approaches a gaussian with width $\mu\sqrt{x}$, proportional to \sqrt{x} , as in [18]. In the classical limit, neglecting $\frac{p_T^2}{p_z^2}$ and the power dependance on

x and p_T , by integrating first in x with the gaussian approximation for the exponential, one gets the behavior $\exp(\frac{-2p_T}{\mu\sqrt{\bar{x}}})$ [19], with an "effective temperature" $\frac{\mu\sqrt{\bar{x}}}{2} = 49(MeV)$ smaller than the range $120 - 150(MeV)$ proposed in [20], but the effect of quantum statistics leads to a harder spectrum for p_T , as it happens for x , since the local maximum for $xq(x)$ for the valence partons is larger than \bar{x} , as it is shown in figures 1 and 2. In fact for the non diffractive contribution of the valence partons, which mainly contribute to the larger p_T , we have:

$$(\overline{p_T})_u = 97(MeV)$$

$$(\overline{p_T})_d = 81(MeV)$$

both larger than $49(MeV)$.

It is worth to stress the attractive feature of the quantum statistical distributions of fixing the free parameters in regions of x , where data have a larger statistics and small systematic errors. In fact, while A' , \bar{A}' and A_G are constrained by eqs.(7a-7b) and by the condition that parton carry the proton momentum, \bar{A} and \bar{b} are fixed by the measurements at small x , \bar{x} , the \tilde{X} 's and the \tilde{Y} 's are fixed by the comparison with the intermediate region, where the valence quarks dominate, and determine the normalization and the Boltzmann behaviour $\exp(-x/\bar{x})$, where the data are scarce and in particular $F_2^n(x)$ is difficult to extract from $F_2^d(x)$ as a consequence of the Fermi motion. Also the disentangling of the q and \bar{q} contributions to the e. m. DIS, which for the unpolarized distributions is achieved with the help of eqs. (7a-7b), implied by the equilibrium conditions (4) and (5) brings to the prediction of a positive (negative) $\Delta\bar{u}(x)$ ($\Delta\bar{d}(x)$) in agreement [11] with the measurement of W^\pm at RHIC. Also the exponential behaviour of the isospin and spin asymmetries of the sea with the same slope of the high x behaviour of the valence partons and of the gluons provides a good test for the statistical distributions.

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